**Math 604 – AP Calculus BC** **Name______________________________**

**Packet I: Operations on Power Series (Term-by-Term Differentiation and Integration) (9.5)**

**Theorem:** Suppose that $S(x)$ is the sum of a **power series** on an interval $I$, that is,

$$S(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

Then, if $x$ is interior to $I$,

i) $$S'(x) = \sum_{n=0}^{\infty} D_x(a_n x^n) = \sum_{n=1}^{\infty} n a_n x^{n-1} = a_1 + 2a_2 x + 3a_3 x^2 + \cdots$$

ii) $$\int_0^x S(t)dt = \sum_{n=0}^{\infty} \int_0^x a_n t^n dt = \sum_{n=0}^{\infty} \frac{a_n}{n+1} x^{n+1} = a_0 x + \frac{1}{2}a_1 x^2 + \frac{1}{3}a_2 x^3 + \frac{1}{4}a_3 x^4 + \cdots$$

This theorem asserts that $S$ is both differentiable and integrable and shows how the derivative and integral may be calculated. It also implies that the radius of convergence of both series is the same as the original series (though it says nothing about the endpoints).

**Example**: Apply the theorem to the geometric series $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$ over $(-1, 1)$ to obtain formulas for two new series.

**Solution:** Differentiating term by term yields

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \cdots \qquad \text{over } (-1, 1)$$

Integrating term by term gives

$$\int_0^x \frac{1}{1-t} dt = \int_0^x 1dt + \int_0^x tdt + \int_0^x t^2 dt + \cdots$$

$$-\ln(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots \qquad \text{over } (-1, 1)$$

If we replace $x$ by $-x$ and multiply both sides by $-1$, we obtain

$$\boxed{\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \qquad \text{over } (-1, 1)}$$

Find the power series representation for $f(x)$ and specify the radius of convergence. Each is somehow related to a geometric series.

| | |
|---|---|
| 1. $f(x) = \frac{1}{1+x}$ | 2. $f(x) = \frac{1}{(1+x)^2}$ <br><br> *Hint: Differentiate problem 1.* |

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|---|---|
| 3. $f(x) = \dfrac{1}{(1-x)^3}$ | 4. $f(x) = \dfrac{x}{(1+x)^2}$ |

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|---|---|
| 5. $f(x)=\dfrac{1}{2-3x}=\dfrac{\frac{1}{2}}{1-\frac{3}{2}x}$ | 6. $f(x)=\dfrac{1}{3+2x}$ |

| | |
|---|---|
| 7. $f(x)=\dfrac{x^2}{1-x^4}$ | 8. $f(x)=\dfrac{x^3}{2-x^3}$ |