Math 604 – AP Calculus BC **Name**_____

Packet H: Power Series and Convergence Sets (Section 9.5)

A power series in *x* has the form:

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

We call the set on which a power series converges its **convergence set.**

The convergence set for a power series $\sum_{n=0}^{\infty} a_n x^n$ is always an **interval** of one of the following three types.

i) The single point $x = 0$.
ii) An interval $(-R, R)$, plus possibly one or both endpoints.
iii) The whole real number line $(-\infty, \infty)$.

The **radius of convergence** is 0, R, and ∞, respectively.

Theorem:
A power series $\sum_{n=0}^{\infty} a_n x^n$ converges absolutely on the interior of its **interval of convergence**.

A power series in *x – a* has the form:

$$\sum_{n=0}^{\infty} a_n (x-a)^n = a_0 + a_1 (x-a) + a_2 (x-a)^2 + a_3 (x-a)^3 + \cdots$$

The convergence set for a power series $\sum_{n=0}^{\infty} a_n (x-a)^n$ is always an **interval** of one of the following three types.

i) The single point $x = a$.
ii) An interval $(a-R, a+R)$, plus possibly one or both endpoints.
iii) The whole real number line $(-\infty, \infty)$.

Find the convergence set of the given power series. *Hint*: First find a formula for the *n*th term, then use the Absolute Ratio Test.

1. $\frac{x}{1\cdot 2} - \frac{x^2}{2\cdot 3} + \frac{x^3}{3\cdot 4} - \frac{x^4}{4\cdot 5} + \frac{x^5}{5\cdot 6} - \cdots$

2. $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$

3. $x + 2x^2 + 3x^3 + 4x^4 + \cdots$

4. $1 - x + \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} - \cdots$

5. $1-\frac{x}{1\cdot 3}+\frac{x^2}{2\cdot 4}-\frac{x^3}{3\cdot 5}+\frac{x^4}{4\cdot 6}-\cdots$

6. $1-\frac{x}{2}+\frac{x^2}{2^2}-\frac{x^3}{2^3}+\frac{x^4}{2^4}-\cdots$

7. $1 + 2x + \frac{2^2 x^2}{2!} + \frac{2^3 x^3}{3!} + \frac{2^4 x^4}{4!} + \cdots$	8. $\frac{(x-1)}{1} + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} + \frac{(x-1)^4}{4} + \cdots$

9. $1+\frac{(x+1)}{2}+\frac{(x+1)^2}{2^2}+\frac{(x+1)^3}{2^3}+\cdots$

10. $\frac{(x+5)}{1\cdot 2}+\frac{(x+5)^2}{2\cdot 3}+\frac{(x+5)^3}{3\cdot 4}+\frac{(x+5)^4}{4\cdot 5}+\cdots$