Math 604 – AP Calculus BC **Name_____________________________**

Packet G: Alternating Series, Conditional, and Absolute Convergence (Section 9.5)

VIII. Alternating-Series Test. Let $a_1 - a_2 + a_3 - a_4 + \cdots$ be an alternating series.

i) If $a_n > a_{n+1} > 0$ (the sequence $\{a_n\}$ is strictly decreasing) and
ii) If $\lim_{n\to\infty} a_n = 0$,

then the series converges.

The absolute value of the remainder (error) R_n involved in approximating the sum S by S_n is less than or equal to the first neglected term.

$$|R_n| = |S - S_n| \le a_{n+1}$$

IX. Absolute Convergence Test.

If $\sum_{n=1}^{\infty} |u_n|$ converges, then $\sum_{n=1}^{\infty} u_n$ converges.

A series $\sum_{n=1}^{\infty} u_n$ is said to **absolutely converge** if $\sum_{n=1}^{\infty} |u_n|$ converges.

X. Absolute Ratio Test. Let $\sum_{n=1}^{\infty} u_n$ be a series of nonzero terms and suppose $\lim_{n\to\infty} \frac{|u_{n+1}|}{|u_n|} = \rho$.

i) If $\rho < 1$, the series converges absolutely (hence converges).
ii) If $\rho > 1$, the series diverges.
iii) If $\rho = 1$, the test in inconclusive.

A series $\sum_{n=1}^{\infty} u_n$ is called **conditionally convergent** if $\sum_{n=1}^{\infty} u_n$ converges but $\sum_{n=1}^{\infty} |u_n|$ diverges. The alternating harmonic series is the premier example of a conditionally convergent series.

$\sum_{n=1}^{n} (-1)^{n+1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ converges but $\sum_{n=1}^{n} \left|(-1)^{n+1} \frac{1}{n}\right| = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ diverges.

Show that each of the alternating series converges and then estimate the error made by using the partial sum S_9 as an approximation to the sum S of the series.

1. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{3n+1}$	2. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}}$
3. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\ln(n+1)}$	4. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\ln n}{n}$

Show that each series converges absolutely.

5. $\sum_{n=1}^{\infty}\left(-\frac{3}{4}\right)^{n}$	6. $\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{n\sqrt{n}}$
7. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{2^{n}}$	8. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^{2}}{e^{n}}$

Classify the series as absolutely convergent, conditionally convergent, or divergent.

9. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{5n}$	10. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{10n+1}$
11. $\sum_{n=2}^{\infty}(-1)^{n}\frac{1}{n\ln n}$	12. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^4}{2^n}$

13. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^2+1}$

14. $\sum_{n=1}^{\infty}\frac{\cos n\pi}{n}$

15. $\sum_{n=1}^{\infty}\frac{\cos n\pi}{n\sqrt{n}}$

16. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n(n+1)}}$

17. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n+1}+\sqrt{n}}$	18. $\sum_{n=1}^{\infty}(-3)^{n+1}\frac{1}{n^2}$
19. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{10n^{1.1}+1}$	20. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n-1}{n}$