Math 604 – AP Calculus BC **Name**_____

Packet F: Positive Series - Other Tests (corresponds to Section 9.4)

V. Direct Comparison Test. Suppose $0 \le a_n \le b_n$ for $n \ge N$.

i) If $\sum_{k=1}^{\infty} b_n$ converges, then so does $\sum_{k=1}^{\infty} a_n$.

ii) If $\sum_{k=1}^{\infty} a_n$ diverges, then so does $\sum_{k=1}^{\infty} b_n$.

VI. Limit Comparison Test. Suppose $a_n \ge 0$, $b_n > 0$, and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$.

i) If $0 < L < \infty$, then $\sum_{k=1}^{\infty} a_n$ and $\sum_{k=1}^{\infty} b_n$ converge or diverge together.

ii) If $L = 0$ and $\sum_{k=1}^{\infty} b_n$ converges, then $\sum_{k=1}^{\infty} a_n$ converges.

iIi) If $L = \infty$ and $\sum_{k=1}^{\infty} b_n$ diverges, then $\sum_{k=1}^{\infty} a_n$ diverges.

VII. Ratio Test. Let $\sum_{k=1}^{\infty} a_n$ be a series of positive terms and suppose $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho$.

i) If $\rho < 1$, the series converges.

ii) If $\rho > 1$, the series diverges.

iii) If $\rho = 1$, the test in inconclusive.

SUMMARY. To test a series $\sum_{k=1}^{\infty} a_n$ of positive terms for convergence or divergence, look carefully at a_n.

1. If $\lim_{n\to\infty} a_n \ne 0$, conclude from the *n*th-Term Test that the series diverges.
2. If a_n involves $n!$, r^n, or n^n try the Ratio Test.
3. If a_n involves only constant powers of *n*, try the Limit Comparison Test. In particular, if a_n is a rational expression in *n*, use this test with b_n as the quotient of the leading terms from numerator and denominator.
4. If the tests above do not work, try the Direct Comparison Test or the Integral Test.
5. Some series require a clever manipulation or a neat trick to determine convergence or divergence.

Use the Limit Comparison Test to determine convergence or divergence.

1. $\sum_{n=1}^{\infty} \frac{n}{n^2 + 2n + 3}$	2. $\sum_{n=1}^{\infty} \frac{3n + 1}{n^3 - 4}$
3. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}}$	4. $\sum_{n=1}^{\infty} \frac{\sqrt{2n+1}}{n^2}$

Use the Ratio Test to determine convergence or divergence.

5. $\sum_{n=1}^{\infty} \frac{8^n}{n!}$	6. $\sum_{n=1}^{\infty} \frac{5^n}{n^5}$
7. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$	8. $\sum_{n=1}^{\infty} \frac{n^3}{(2n)!}$

Determine convergence or divergence for each of the series. Indicate the test you used.

9. $\sum_{n=1}^{\infty} \frac{n}{n+200}$	10. $\sum_{n=1}^{\infty} \frac{n+3}{n^2\sqrt{n}}$
11. $\sum_{n=1}^{\infty} \frac{n^2}{n!}$	12. $\sum_{n=1}^{\infty} \frac{4n^3+3n}{n^5-4n^2+1}$

13. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\cdots$

14. $\frac{2}{1\cdot 3\cdot 4}+\frac{3}{2\cdot 4\cdot 5}+\frac{4}{3\cdot 5\cdot 6}+\frac{5}{4\cdot 6\cdot 7}+\cdots$

15. $\frac{1}{3}+\frac{2}{3^2}+\frac{3}{3^3}+\frac{4}{3^4}+\cdots$

16. $1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\cdots$

17. $\sum_{n=1}^{\infty} \frac{1}{2+\sin^2 n}$

18. $\sum_{n=1}^{\infty} \frac{4+\cos n}{n^3}$

19. $\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$

20. $\sum_{n=1}^{\infty} \frac{4^n+n}{n!}$