**Math 604 – AP Calculus BC** **Name_____________________________**

**Packet E: Positive Series - Integral Test (corresponds to Section 9.4)**

**II. Integral Test.** Let $f$ be a continuous, positive, nonincreasing function on the interval $[1, \infty)$ and suppose $a_k = f(k)$ for al positive integers $k$.

Then the infinite series

$$\sum_{k=1}^{\infty} a_k$$

converges if and only if the improper integral

$$\int_1^{\infty} f(x)dx$$

converges.

The series $\sum_{k=1}^{\infty} a_k$ and the improper integral $\int_1^{\infty} f(x)dx$ converge or diverge together.

**III. *p*-Series Test.** The series

$$\sum_{k=1}^{\infty} \frac{1}{k^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots$$

where $p$ is a constant, is called a ***p*-series**.

a) If $p > 1$, the $p$-series converges.
b) If $p \leq 1$, the $p$-series diverges.

**IV. *n*th-Term for Divergence.**

a) If the series $\sum_{k=1}^{\infty} a_k$ converges, then $\lim_{k \to \infty} a_k = 0$.

b) If $\lim_{k \to \infty} a_k \neq 0$ or does not exist, then the series $\sum_{k=1}^{\infty} a_k$ diverges.

Use the Integral Test test to decide about the convergence or divergence of the series

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|---|---|
| 1. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k+2}$ | 2. $\displaystyle\sum_{k=1}^{\infty} \frac{k}{1+k^2}$ |
| 3. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k+2}}$ | 4. $\displaystyle\sum_{k=1}^{\infty} \frac{2}{1+k^2}$ |
| 5. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{10k+3}$ | 6. $\displaystyle\sum_{k=1}^{\infty} \frac{k}{e^k}$ |

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| 7. $\sum_{k=1}^{\infty} \frac{1}{(4+3k)^{3/2}}$ | 8. $\sum_{k=1}^{\infty} \frac{k^2}{1+k^3}$ |
| 9. $\sum_{k=1}^{\infty} ke^{-k^2}$ | 10. $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$ |

Use any test developed so far to decide about the convergence or divergence of the series. Give a reason for your conclusion.

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| 11. $\sum_{k=1}^{\infty} \frac{k^2+1}{k^2+5}$ | 12. $\sum_{k=1}^{\infty} \left(\frac{3}{\pi}\right)^k$ |

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| 13. $\sum_{k=1}^{\infty}\left[\left(\frac{1}{2}\right)^k + \frac{k-1}{2k+1}\right]$ | 14. $\sum_{k=1}^{\infty}\left(\frac{1}{k^2} + \frac{1}{2^k}\right)$ |
| 15. $\sum_{k=1}^{\infty}\sin\left(\frac{k\pi}{2}\right)$ | 16. $\sum_{k=1}^{\infty} k\sin\frac{1}{k}$ |
| 17. $\sum_{k=1}^{\infty} k^2 e^{-k}$ | 18. $\sum_{k=2}^{\infty}\left(\frac{1}{k} - \frac{1}{k-1}\right)$ |

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| 19. $\displaystyle\sum_{k=1}^{\infty} \frac{\tan^{-1} k}{1+k^2}$ | 20. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{1+4k^2}$ |

21. For what values of $p$ does $\displaystyle\sum_{k=3}^{\infty} 1/[n(\ln n)^p]$ converge?

22. Does $\displaystyle\sum_{k=3}^{\infty} 1/[n \cdot \ln n \cdot \ln(\ln n)]$ converge or diverge?