Math 604 – AP Calculus BC **Name______________________________**

Packet D: Infinite Series (corresponds to Section 9.1)

I. Geometric Series. A series of the form $\sum_{k=1}^{\infty} ar^{k-1} = a + ar + ar^2 + ar^3 + \cdots$ where $a \neq 0$ is called a **geometric series**. This series converges with sum $S = a/(1-r)$ if $|r| < 1$, but diverges if $|r| \geq 1$.

Indicate whether the given series converges or diverges. If it converges, find its sum. *Hint*: it may help you to write out the first few terms of the series.

1. $\sum_{k=1}^{\infty} \left(\frac{1}{5}\right)^k$	2. $\sum_{k=0}^{\infty} \left[2\left(\frac{1}{3}\right)^k + 3\left(\frac{1}{6}\right)^k\right]$
3. $\sum_{k=1}^{\infty} \frac{k-3}{k}$	4. $\sum_{k=1}^{\infty} \left(\frac{4}{3}\right)^k$

5. $\sum_{k=1}^{\infty} \frac{k!}{10^k}$	6. $\sum_{k=3}^{\infty} \left(\frac{e}{\pi}\right)^{k-1}$
7. $\sum_{k=1}^{\infty} \left(\frac{3}{(k+1)^2} - \frac{3}{k^2}\right)$	8. $\sum_{k=4}^{\infty} \frac{4}{k-3}$

COLLAPSING SERIES. A geometric series is one of the few series where we can actually calculate S_n; a collapsing series is another.

Example. Show that the following series converges and find its sum.

$$\sum_{k=1}^{\infty}\left(\frac{1}{(k+2)(k+3)}\right)$$

Solution. Use a partial fraction decomposition to write

$$\frac{1}{(k+2)(k+3)}=\frac{1}{k+2}-\frac{1}{k+3}$$

Then,

$$S_n=\sum_{k=1}^{n}\left(\frac{1}{k+2}-\frac{1}{k+3}\right)=\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\cdots+\left(\frac{1}{n+2}-\frac{1}{n+3}\right)$$

$$=\frac{1}{3}-\frac{1}{n+3}$$

Therefore,

$$\lim_{n\to\infty}S_n=\frac{1}{3}$$

Use the above example to determine whether the given series converges or diverges. If it converges, find its sum.

9. $\sum_{k=2}^{\infty}\left(\frac{2}{k(k-1)}\right)$	10. $\sum_{k=1}^{\infty}\left(\frac{4}{(4k-3)(4k+1)}\right)$

11. $\sum_{k=1}^{\infty}\left(\frac{6}{(2k-1)(2k+1)}\right)$	12. $\sum_{k=1}^{\infty}\left(\frac{40k}{(2k-1)^2(2k+1)^2}\right)$
13. $\sum_{k=1}^{\infty}\left(\frac{2k+1}{k^2(k+1)^2}\right)$	14. $\sum_{k=1}^{\infty}\left(\frac{1}{\ln(k+2)}-\frac{1}{\ln(k+1)}\right)$
15. $\sum_{k=1}^{\infty}\left(\tan^{-1}(k)-\tan^{-1}(k+1)\right)$	16. $\sum_{k=1}^{\infty}\left(\frac{2^{2k}}{(2k)!}-\frac{2^{2k+1}}{(2k+1)!}\right)$