

**Packet C: Taylor & Maclaurin Polynomial Practice (Sections 9.2 & 9.3)**

**Maclaurin Series** at  $c = 0$ :  $f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(n)}(0)}{n!}x^n + \dots = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!}x^k$

Use the table of Maclaurin series. Construct the first three nonzero terms and the general term of the Maclaurin series generated by the function, and give the interval of convergence.

1.  $f(x) = \sin 2x$

2.  $f(x) = \tan^{-1} x^2$

3.  $f(x) = e^{-2x}$

**Taylor Series** at  $x = c$  :

$$f(x) \approx f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \dots = \sum_{k=0}^{\infty} \frac{f^{(k)}(c)}{k!}(x-c)^k$$

Find the Taylor Polynomials of order 3 centered at the given points.

4.  $f(x) = \frac{1}{x+1}$  ;  $c = 2$

5.  $f(x) = e^{x/2}$  ;  $c = 1$

6. Let  $f$  be a function that has derivatives of all orders for all real numbers. Assume that

$$f(1) = 4$$

$$f'(1) = -1$$

$$f''(1) = 3$$

$$f'''(1) = 2$$

a) Write the third order Taylor polynomial for  $f$  at  $c = 1$  and use it to approximate  $f(1.2)$ .

b) Write the second order Taylor polynomial for  $f'$ , the derivative of  $f$ , at  $c = 1$  and use it to approximate  $f'(1.2)$ .

**Taylor Formula with Remainder** at  $x = c$  :

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n + R_n(x)$$

where the remainder (error) is  $R_n(x) = \frac{f^{(n+1)}(a)}{(n+1)!}(x-c)^{n+1}$  and  $a$  is some point between  $x$  and  $c$ .

Find a good estimate for the maximum value of the given expression,  $a$ , being in the given interval.  
Answers may vary depending upon the technique used.

7. $ \tan a + \sec a ; [-\frac{\pi}{4}, 0]$	8. $\left  \frac{4}{a+2} \right ; [-1.5, 2]$
9. $\left  \frac{\sin a}{a+1} \right ; [0, 1]$	10. $\left  \frac{a^4 - a}{\sin a} \right ; [1, 3]$

For 11 and 12, find a formula for  $R_6(x)$ , the remainder for the Taylor polynomial of order 6 based at  $c$ . Then estimate  $|R_6(0.5)|$ , that is, give a good upper bound for it.

11.  $e^{-x}; c=0$

12.  $\frac{1}{x-2}; c=1$

13. Find the third order Maclaurin polynomial for  $(1 + x)^{3/2}$  and estimate the error  $R_3(x)$  if  $-0.1 \leq x \leq 0$ .

14. Note that the fourth-order Maclaurin polynomial for  $\sin x$  is really of third degree since the coefficient of  $x^4$  is 0. Thus

$$\sin x = x - \frac{x^3}{6} + R_4(x)$$

Show that if  $0 \leq x \leq 0.5$ ,  $|R_4(x)| \leq 0.0002605$ . Use this result to approximate  $\int_0^{0.5} \sin x \, dx$  and give an estimate of the error.